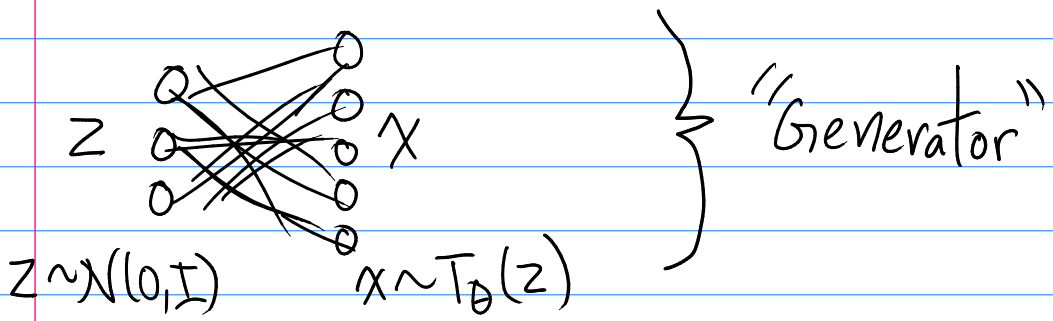
Generative Adversarial Networks (GANs)

Given $X_1, \ldots, X_n \sim P$, generate more data from P

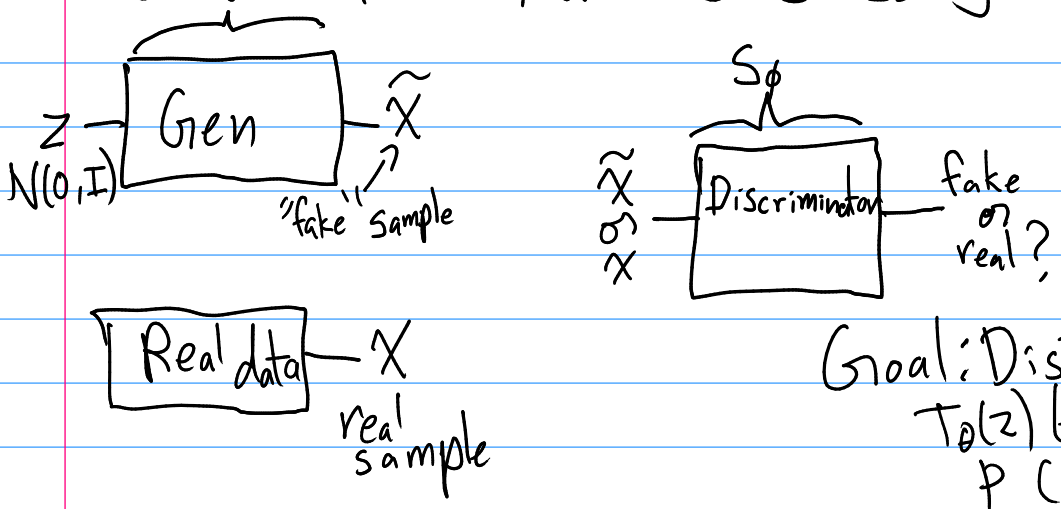
Problem: P may be complex

Soln: Use a NN to map samples from $N(0, I)$ to samples from P.
i.e. if $Z \sim N(0,I)$, then $T_\theta(z) \sim P$



How to make this work?
- VAE: Enforce that we could encode + decode
- GANs: Make sure samples from $T_\theta(z)$ are indistinguishable from true data.

Use another NN to classify real vs fake



Goal: Distinguish between $T_\theta(z)$ (fake samples) vs P (real samples)

Deriving the loss

Fenchel Conjugate.

Let $f(x): \mathbb{R} \to \mathbb{R}$ be some function.

Fenchel conjugate of f: $f^*(x) = \max_y (xy - f(y))$

Example: $f(x) = \boxed{x \log x}$

$f^*(x) = \max_y [xy - y\log y]$. $\frac{d}{dy}[xy - y\log y] = x - \log y - 1 = 0$

$\log y = x - 1 \Rightarrow y = \exp(x-1)$

$f^*(x) = x\exp(x-1) - (x-1)\exp(x-1) = \exp(x-1)$

$f^{**}(x) = \max_y [xy - \exp(y-1)]$. $\frac{d}{dy}[xy - \exp(y-1)] = x - \exp(y-1) = 0$

$\log x = y - 1$

$y = 1 + \log x$

$\hookrightarrow = x(1+\log x) - \exp(1+\log x - 1)$

$= x + x\log x - x = \boxed{x \log x}$

Claim: f is convex iff $f^{**} = f$.

F-divergences

$D_f(p\|q) = \int q(x) f\left(\frac{p(x)}{q(x)}\right) dx$, where f is strictly convex and $f(1) = 0$.

Example: $f(t) = t\log t$. $D_f(p\|q) = \int q(x) \cdot \frac{p(x)}{q(x)} \log\left(\frac{p(x)}{q(x)}\right) dx$

$= \int p(x) \log\left(\frac{p(x)}{q(x)}\right) dx$

$\triangleq KL(p(x)\|q(x))$

Claim: $D_f(p\|q) \geq 0$, $= 0$ if $p = q$.

$$\int q(x) f\left(\frac{p(x)}{q(x)}\right) dx \geq f\left(\int q(x) \frac{p(x)}{q(x)} dx\right) = f(1) = 0$$

Jensen's ineq.

$$\int q(x) f(1)\, dx = \int 0\, dx = 0$$

Back to GANs

Goal: Get density $q_\theta \approx p$

Goal: $\min_\theta D_f(p(x)\|q_\theta(x))$ (for some f)

$T_\theta(z)$: $q_\theta(x)$ is density fn at x of RV $z \sim N(0, I)$, and output $T_\theta(z)$

$$D_f(p(x)\|q_\theta(x)) = \int q(x) f\left(\frac{p(x)}{q(x)}\right) dx$$

$$= \int q(x) \left[\max_{s(x) \in \mathbb{R}} s(x)\frac{p(x)}{q(x)} - f^*(s(x))\right] dx \qquad (f = f^{**})$$

$$f^{**}(t) = \max_s st - f^*(s)$$

$$= \max_{S: \mathbb{R}^d \to \mathbb{R}} \left[\int p(x) s(x)\, dx - \int q(x) f^*(s(x))\, dx\right]$$

$$= \max_{S: \mathbb{R}^d \to \mathbb{R}} \mathop{E}_{x \sim p(x)}[S(x)] - \mathop{E}_{x \sim q_\theta(x)}[f^*(s(x))]$$

$$\min_\theta D_f(p(x)\|q_\theta(x)) \approx \min_\theta \max_\phi \left[\int p(x) S_\phi(x)\, dx - \int q_\theta(x) f^*(S_\phi(x))\, dx\right]$$

$$\approx \min_\theta \max_\phi \left[\frac{1}{n}\sum_{i=1}^n S_\phi(x_i) - \frac{1}{m}\sum_{j=1}^m f^*(S_\phi(T_\theta(z_j)))\right]$$

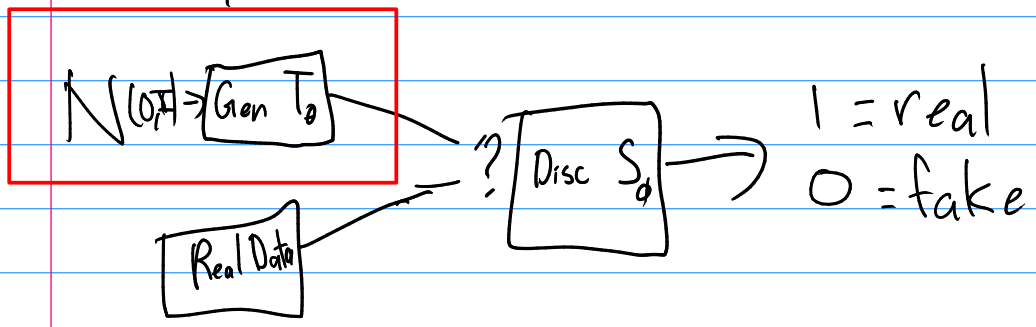
- $\min_\theta$: Generator
- $\max_\phi$: Discriminator
- x_i: Real data
- $T_\theta(z_j)$: Fake data, $z_j \sim N(0, I)$, $j = 1$ to m.

Jensen-Shannon GAN: $D_{JS}(p\|q) = KL(p\|\frac{p+q}{2}) + KL(q\|\frac{p+q}{2})$

$f^*(t) = -\log(1-\exp(t)) - \log 4$

Also, reparameterize $S \leftarrow \log S$

$$\min_{T_\theta} \max_{S_\phi} \frac{1}{n}\sum \log S_\phi(x_i) + \frac{1}{m}\sum \log\left(1 - S_\phi(T_\theta(z_j))\right)$$



Optimization?

Simultaneously update ϕ and θ using gradients

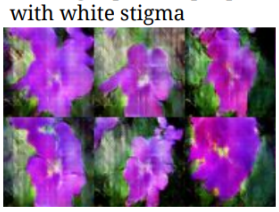
$$\phi^{(t+1)} \leftarrow \phi^{(t)} + \eta_\phi \nabla_\phi \left[\frac{1}{n}\sum \log S_{\phi^{(t)}}(x_i) + \frac{1}{m}\sum \log\left(1 - S_{\phi^{(t)}}\left(T_{\theta^{(t)}}(z_j)\right)\right)\right]$$

$$\theta^{(t+1)} \leftarrow \theta^{(t)} - \eta_\theta \nabla_\theta \left[\frac{1}{m}\sum \log\left(1 - S_{\phi^{(t)}}\left(T_{\theta^{(t)}}(z_j)\right)\right)\right]$$

this small bird has a pink breast and crown, and black primaries and secondaries.

this magnificent fellow is almost all black with a red crest, and white cheek patch.

the flower has petals that are bright pinkish purple with white stigma



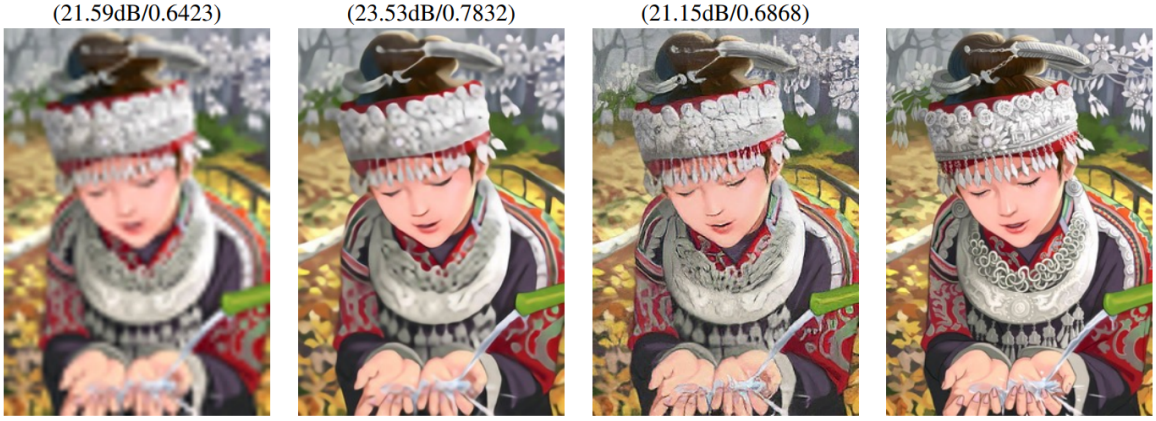
this white and yellow flower have thin white petals and a round yellow stamen

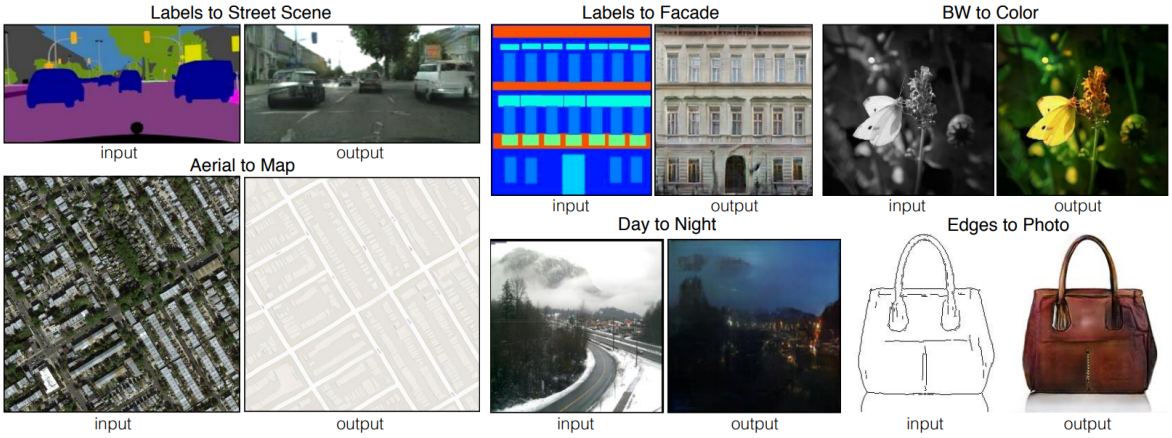
bicubic (21.59dB/0.6423)

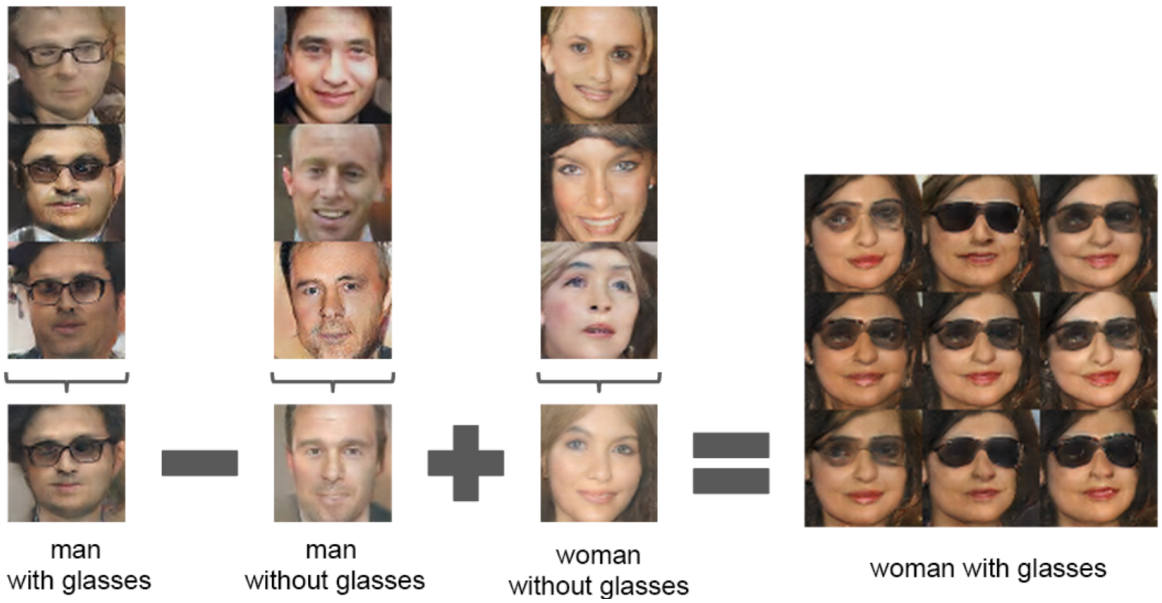
SRResNet (23.53dB/0.7832)

SRGAN (21.15dB/0.6868)

original




Labels to Street Scene
input
output
Labels to Facade
input
output
BW to Color
input
output
Aerial to Map
input
output
Day to Night
input
output
Edges to Photo
input
output


man
with glasses
man
without glasses
woman
without glasses
woman with glasses